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Shell evolution : A paradigm of structure of exotic nuclei ?

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The evolution of shell structure and magic numbers of exotic nuclei are discussed with a rather pedagogical introduction. A major origin of the shell evolution is shown to be the spin-isospin dependent central part of the nucleon-nucleon interaction in nuclei. The importance and robustness of this mechanism is shown in connection to the $\tau \cdot \tau\ \sigma \cdot \sigma$ interaction. In neutron-rich exotic nuclei, magic numbers such as N=8, 20, *etc.* can disappear, while N=6, 16, *etc.* arise. The $\tau \cdot \tau\ \sigma \cdot \sigma$ interaction should be related to Gamov-Teller and magnetic properties. Another mechanism of the shell evolution is shown to be the tensor interaction.

1. INTRODUCTION

I shall discuss, in this talk, our recent studies [1,2] on the shell structure of exotic nuclei, indicating that the shell, or magic, structure can be varied in going from stable to exotic nuclei and such change is strongly related to certain properties of the nucleon-nucleon interaction. I shall propose a paradigm of shell evolution as one of the key principles in determining structure of exotic nuclei.

The magic numbers play a key role in many-body physics as the most fundamental quantity reflecting possible shell structure. The nuclear shell model has been started by Mayer and Jensen by identifying its magic numbers and their origin [3]. The study of nuclear structure has been advanced on the basis of the shell structure thus clarified. The shell-model studies, on the other hand, have been made predominantly for stable nuclei, which are on or near the β-stability line in the nuclear chart. This is basically because only those nuclei have been accessible experimentally. In such stable nuclei, the magic numbers suggested by Mayer and Jensen remain valid, and the shell structure can be understood well in terms of the harmonic oscillator potential with a spin-orbit splitting.

Recently, studies on exotic nuclei far from the β-stability line have started owing to development of radioactive nuclear beams, as discussed extensively in this conference. The magic numbers in such exotic nuclei can be a quite intriguing issue. We shall show that new magic numbers appear and some other conventional ones disappear in moving from stable to exotic nuclei in a rather novel manner due to a particular part of the nucleon-nucleon interaction [1,2]. Although the magic numbers are prominent features,

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there are gradual changes of underlying single-particle or shell structure. Including those changes, I would like to call these phenomena "Shell Evolution".

If single-particle energies are calculated by the Woods-Saxon potential, they change as the proton number (Z) or the neutron number (N) varies. In this case, the single-particle energies are shifted basically in parallel, keeping their relative energies (or mutual differences of the energies) almost unchanged. This kind of change is due to the variation of the potential radius depending on $A(= N + Z)$ and/or the shift of the potential depth associated with N/Z asymmetry, and is not called Shell Evolution. Note that, even with the Woods-Saxon potential, the relative energies can be changed near drip lines owing to varying influences of the centrifugal potential, but such changes are not the subject of this talk, and may be referred better with a different nomenclature because of its kinetic origin.

The Shell Evolution means that the relative energies can vary rather significantly as N and/or Z changes. If this energy change becomes sufficiently significant, even the shell gap can disappear or a new gap may arise. Thus, as a result of the Shell Evolution, the magic numbers may change.

The Shell Evolution has been seen in the p-shell and sd-shell already [1]. In order to understand it, we use effective single-particle energies as explained in sect. 2. The Shell Evolution seem to occur, in many cases, due to the common mechanism related to the Nucleon-Nucleon (NN) interaction as discussed in sect. 3. Because of this generality and robustness, one can raise the paradigm of the Shell Evolution as an underlying principle determining the structure of exotic nuclei. I mention briefly how mean field theories can be improved along this line.

2. EFFECTIVE SINGLE-PARTICLE ENERGIES

In order to understand underlying single-particle properties of a nucleus, we can make use of *effective (spherical) single-particle energies (ESPE's)*, which represent mean effects from the other nucleons on a nucleon in a specified single-particle orbit.

In the shell model, single-particle orbits are classified into two groups. One is for those in the inert core, which is a closed shell. The other is for the orbits outside the inert core, and valence nucleons are moving on those valence orbits. Here, we are discussing spherical single-particle orbits with good orbital and total angular momenta, l and j. Each orbit has its single-particle energy. This energy contains the kinetic energy and the binding from the nucleons in the inert core. For nuclei consisting of the inert core and one more nucleon (like ^{17}O), these single-particle energies give the energy levels of the nucleus unless the inert core is broken. In the usual shell model calculations, these single-particle energies are called *bare* single-particle energies.

As one adds more valence nucleons on top of the inert core, effects of so-called residual interaction becomes larger. The shell-model (residual) interaction between valence nucleons includes various multipole components. We shall discuss on them. The quadrupole component, for instance, is the origin of the quadrupole collectivity producing vibrational and rotational spectra. To be intuitive, the quadrupole component produces more binding energy, when the relative angle (with respect to the center of the nucleus) between two interacting nucleons are smaller, i.e., two nucleons are correlated in angle. By making such correlation collective, one comes to a deformed shape. For instance, in the case of a prolate shape, valence nucleons are gathered near the longer axis. In contrast, in the monopole component, effects depending on this relative angle between are averaged out. Namely, the monopole component does not care how far the two interacting nucleons are in angle.

We next discuss how to calculate the monopole component. The two-body matrix element of the interaction depends on the angular momentum J, coupled by the two interacting nucleons in orbits j_1 and j_2. This J-dependence is averaged out with a weight factor $(2J+1)$. Since mean effects are being considered, only diagonal matrix elements are taken. The monopole interaction is thus obtained with a matrix element [4,5]:

$$V^T_{j_1 j_2} = \frac{\sum_J (2J+1) < j_1 j_2 | V | j_1 j_2 >_{JT}}{\sum_J (2J+1)}, \quad \text{for } T = 0,\, 1, \tag{1}$$

where $< j_1 j_2 | V | j'_1 j'_2 >_{JT}$ stands for the matrix element of a two-body interaction, V. Although this is still a two-body interaction, it has no dependence on J. Here, the isospin dependence, T=0 or 1, is kept, however.

We point out an important property of the monopole interaction. Since the angular correlation is taken away, two nucleons can be at any magnetic substate, yielding the same binding energy. So, in evaluating its effects in a system with many valence nucleons, only the number of nucleons in each orbit matters. This implies further that the effect of the monopole interaction can be accumulated, and its effect becomes largest as the orbit is fully occupied. On the other hand, this is not the case for other multipoles, and the effect becomes vanished for fully occupied orbit. Mathematically, the monopole operator has a finite trace, whereas the trace is zero for other multipoles. Thus, even weak monopole interaction can be magnified in its effect by a large number of nucleons.

The monopole Hamiltonian consists of the bare single-particle energies stated above and the monopole interaction (between valence nucleons).

The ESPE is evaluated from this monopole Hamiltonian, and naturally can play a role of a measure of mean effects from the other nucleons, including valence ones. For simplicity, The normal filling configuration is used normally. Note once again that, because the J dependence is taken away, only the number of nucleons in each orbit matters. As a natural assumption, the possible lowest isospin coupling is assumed for protons and neutrons in the same orbit. The ESPE of an *occupied* orbit is defined to be the separation energy of this orbit with the opposite sign. Note that the separation energy implies the minimum energy needed to take a nucleon out of this orbit. The ESPE of an *unoccupied* orbit is defined to be the binding energy gain by putting a proton or neutron into this orbit with the opposite sign.

Thus, effective single-particle energies can be defined and we now use them. In actual calculations, the isospin coupling must be considered between the two orbits in the monopole Hamiltonian, but this is a rather theoretical detail and is not discussed here (See eq. (1) of [5] for example).

3. SHELL GAP AT N=16

We now show ESPE's for a stable nucleus ^{30}Si and for an exotic nucleus ^{24}O in Fig. 1 (a) and (b), respectively. The shell model Hamiltonian is the one derived in [5]. This

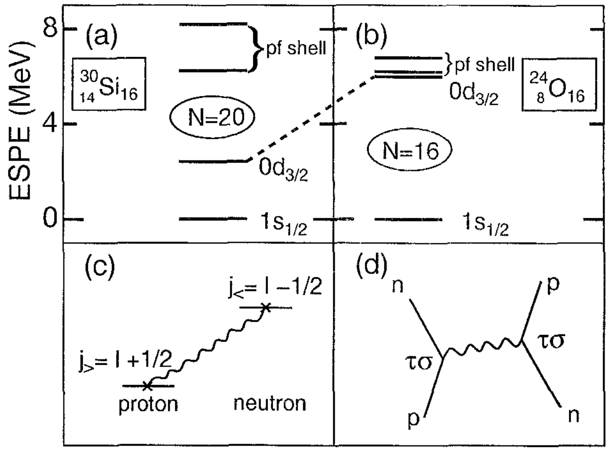

Figure 1. ESPE's for (a) ^{30}Si and (b) ^{24}O, relative to $1s_{1/2}$. The dotted line connecting (a) and (b) is drawn to indicate the change of the $0d_{3/2}$ level. (c) The major interaction producing the change between (a) and (b). (d) The elementary process relevant to the interaction in (c). Taken from [1].

Hamiltonian produces quite good agreement with experiment for a large number of nuclei within a single framework [5,6].

In Fig. 1 (b), shown are ESPE's for ^{24}O, where the $0d_{3/2}$ is lying much higher, very close to the pf shell. A considerable gap ($\sim$ 4 MeV) is between the $0d_{3/2}$ and the pf shell for the stable nucleus ^{30}Si, whereas an even larger gap ($\sim$ 6 MeV) is found between $0d_{3/2}$ and $1s_{1/2}$ for ^{24}O. The basic mechanism of this dramatic change is the strongly attractive interaction shown schematically in Fig. 1 (c), where $j_> = l + 1/2$ and $j_< = l - 1/2$ with l being the orbital angular momentum. In the present case, l=2. One now should remember that valence protons are added into the $0d_{5/2}$ orbit as Z increases from 8 to 14. Due to a strong attraction between a proton in $0d_{5/2}$ and a neutron in $0d_{3/2}$, as more protons are put into $0d_{5/2}$, a neutron in $0d_{3/2}$ is more strongly bound. Thus, the $0d_{3/2}$ ESPE for neutrons is so low in ^{30}Si as compared to that in ^{24}O.

4. SPIN-ISOSPIN DEPENDENCE NN INTERACTION

The process illustrated in Fig. 1 (d) produces the attractive interaction in Fig. 1 (c). The NN interaction in this process is written as

$$V_{\tau\sigma} = \tau \cdot \tau \; \sigma \cdot \sigma \; f_{\tau\sigma}(r). \tag{2}$$

Here, the symbol "·" denotes a scalar product, τ and σ stand for isospin and spin operators, respectively, r implies the distance between two interacting nucleons, and $f_{\tau\sigma}$ is a function of r. In the long range (or no r-dependence) limit of $f_{\tau\sigma}(r)$, the interaction in eq.(2) can couple only a pair of orbits with the same orbital angular momentum l, which are nothing but $j_>$ and $j_<$.

The σ operator couples $j_>$ to $j_<$ (and vice versa) much more strongly than $j_>$ to $j_>$ or $j_<$ to $j_<$. Therefore, the spin flip process is more favored in the vertexes in Fig. 1 (d). The same mathematical mechanism works for isospin: the τ operator favors charge exchange processes. Combining these two properties, $V_{\tau\sigma}$ produces large matrix elements for the spin-flip isospin-flip processes: proton in $j_> \to$ neutron in $j_<$ and vice versa. This gives rise to the interaction in Fig. 1 (c). This feature is a general one and is maintained with $f_{\tau\sigma}(r)$ in eq.(2) with reasonable r dependences.

Although $V_{\tau\sigma}$ yields sizable attraction between a proton in $j_>$ and a neutron also in $j_>$, the effect is weaker than in the case of Fig. 1 (c).

In stable nuclei with $N \sim Z$ with ample occupancy of the $j_>$ orbit in the valence shell, the proton (neutron) $j_<$ orbit is lowered by neutrons (protons) in the $j_>$ orbit. In exotic nuclei, this lowering can be absent, and then the $j_<$ orbit is located rather high, not far from the upper shell. In this sense, the proton-neutron $j_>$-$j_<$ interaction enlarges a gap between major shells for stable nuclei with proper occupancy of relevant orbits.

The origin of the strongly attractive $V_{\tau\sigma}$ is quite clear. The One-Boson-Exchange-Potentials (OBEP) for π and ρ mesons have this type of terms as major contributions. While the OBEP is one of major parts of the effective NN interaction, the effective NN interaction in nuclei can be provided by the G-matrix calculation with core polarization corrections. Such effective NN interaction will be called simply G-matrix interaction for brevity. The G-matrix interaction should maintain the basic features of meson exchange processes, and, in fact, existing G-matrix interactions generally have quite large matrix elements for the cases shown in Fig. 1 (c) [7].

We would like to point out that the $1/N_c$ expansion of QCD by Kaplan and Manohar indicates that $V_{\tau\sigma}$ is one of three leading terms of the NN interaction [8]. Since the next order of this expansion is smaller by a factor $(1/N_c)^2$, the leading terms should have rather distinct significance.

I also point out that the $V_{\tau\sigma}$ interaction is related to spin-isospin properties of nuclei [2]. Along this line, Suzuki gave a talk on the Gamov-Teller and magnetic properties of p-shell nuclei in this conference [9].

5. GAP AT $N = 20$ AND STRUCTURE OF $N = 20$ EXOTIC NUCLEI

We now turn to exotic nuclei with $N \sim 20$. Figs. 2 (a) and (b) show the ESPE's of neutrons for oxygen isotopes and $N = 20$ isotones, respectively [10]. The small effective gap between $0d_{3/2}$ and the pf shell for neutrons is seen in oxygen isotopes in Fig. 2 (a), while this gap becomes wider as Z increases in the $N = 20$ isotones in Fig. 2 (b). This small gap for smaller Z is nothing but what we have seen for ^{24}O in Fig. 1 (b). Thus, the disappearance of N=20 magic structure in exotic nuclei with Z much smaller than 20 and the appearance of the new magic structure in ^{24}O [11] have the same origin. Furthermore, one sees a less pronounced gap between $0d_{5/2}$ and $1s_{1/2}$ at $N = 14$ in Fig. 2 (a). This gap makes ^{22}O a magic-like nucleus [12].

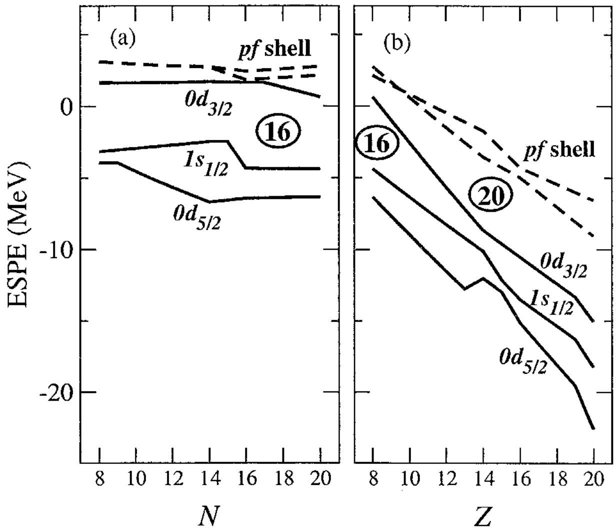

Figure 2. Effective Single Particle Energies (ESPE's) of neutrons for (a) oxygen isotopes from $N = 8$ to 20 and (b) $N = 20$ isotones from $Z = 8$ to 20. Taken from [10].

6. MAGIC NUMBERS IN THE p-SHELL: N=6 VS N=8

A very similar mechanism works for p-shell nuclei. The neutron $0p_{1/2}$ orbit becomes higher as the nucleus loses protons in its spin-flip partner $0p_{3/2}$. The N=8 magic structure then disappears, and N=6 becomes magic, similarly to N=16 magic number in sd shell. As a consequence, ^{8}He is well bound, whereas ^{9}He is not bound. This is analogous to the situation that ^{24}O is well bound, but ^{25}O is unbound.

7. HEAVIER NUCLEI: N=34, *etc.*

Moving back to heavier nuclei, from the strong interaction in Fig. 1 (c), we can predict other magic numbers, for instance, N=34 associated with the $0f_{7/2}$-$0f_{5/2}$ interaction. A recent experiment seems to support N=34 new magic number [13].

8. SUMMARY AND PERSPECTIVES

In summary, we showed how shell structure and magic numbers are changed in nuclei far from the β-stability line: N=6, 16, 34, *etc.* can become magic numbers in neutron-rich exotic nuclei, while usual magic numbers, N=8, 20, 40, *etc.*, may disappear. The mechanism of this change can be explained by the strong attractive $V_{\tau\sigma}$ interaction which has robust origins in OBEP, G-matrix and QCD. In fact, simple structure such as magic numbers should have a simple and robust basis. Including other possible origins, I would like to propose that the structure of stable and exotic nuclei should be studied with the paradigm of Shell Evolution where the shell structure (and magic numbers) can vary significantly as results of variable contributions of nucleon-nucleon interaction and many-body dynamics, depending on Z and N. In fact, Nakada has shown in this conference that the M3Y interaction produces results consistent with this paradigm [14]. As another mechanism of the Shell Evolution, I mention the tensor interaction. The tensor interaction is quite strong. This is the case also from the viewpoint of QCD [8]. The tensor interaction can be shown to produce characteristic and strong effects on ESPE's for the combination of the p-shell and sd-shell orbits [15], ending up with various interesting properties in exotic carbon isotopes [16].

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